

# Noisy News in Business Cycles

Mario Forni

Università di Modena e Reggio Emilia  
CEPR and RECent

Marco Lippi

Università di Roma La Sapienza  
and EIEF

Luca Gambetti\*

Universitat Autònoma de Barcelona  
and Barcelona GSE

Luca Sala<sup>†</sup>

Università Bocconi, IGIER  
and Baffi Center

## Abstract

We investigate the role of “noise” shocks as a source of business cycle fluctuations. To do so we set up a simple model of imperfect information and derive restrictions for identifying the noise shock in a VAR model. The novelty of our approach is that identification is reached by means of *dynamic* rotations of the reduced form residuals. We find that noise shocks generate hump-shaped responses of GDP, consumption and investment and account for about a third of their prediction error variance at business cycle horizons.

JEL classification: C32, E32, E62.

Keywords: Nonfundamentalness, SVAR, Imperfect Information, News, Noise, Signal, Business cycles.

---

\*Corresponding author: Office B3.1130 Departament d’Economia i Historia Econòmica, Edifici B, Universitat Autònoma de Barcelona, Bellaterra 08193, Barcelona, Spain. Tel. +34 935814569; e-mail: luca.gambetti@uab.cat. The financial support from the Spanish Ministry of Science and Innovation through grant ECO2009-09847 and the Barcelona Graduate School Research Network is gratefully acknowledged.

<sup>†</sup>The financial support of MIUR (PRIN grant prot. 2009BL8BEF-001 “Fluttuazioni finanziarie, ciclo economico ed inflazione” and the financial support from the Baffi Center at Università Bocconi are gratefully acknowledged.

# 1 Introduction

There has recently been a renewed interest in the old idea that business cycles could be driven by changes in the expectations about future economic conditions (early references are Pigou, 1927, and Keynes, 1936). The literature has focused mainly on anticipated changes in productivity, the so-called “news shocks” (Cochrane, 1994). The seminal paper by Beaudry and Portier, 2006, (BP henceforth) finds that news shocks account for the bulk of fluctuations in GDP and generate the pattern of comovements among macroeconomic aggregates typically observed over the cycle.<sup>1</sup> Several papers have provided theoretical foundations for these results, by proposing models where news shocks can drive the business cycle (see e.g. Jaimovich and Rebelo, 2009, Den Haan and Kaltenbrunner, 2009, Schmitt-Grohe and Uribe, 2008). Key in these models is that news shocks are assumed to be observable by the agents.

A stream of the literature on news shocks has departed from the assumption of perfect information and proposed models where agents have imperfect information (Sims, 2003, Beaudry and Portier, 2004, Lorenzoni, 2009, Angeletos and La’O, 2010, among others). In the theoretical work by Lorenzoni, 2009, for instance, agents base their optimal decisions on a mixture of a news on aggregate productivity and noise. Though they can eventually disentangle news from noise, their current action can only rely on such a noisy signal. As a consequence agents’ expectations take time to completely adjust, and the final result depends on the size of the noise within the observable signal. In particular, if the signal is just noise, the economy returns to its initial state, whereas if the signal contains productivity news, the economy gradually reaches a new level of activity.

Assuming that agents base their decisions on noisy information seems quite plausible, in particular for events —like improvements in technology— whose effects propagate slowly and therefore are not immediately revealed by observable economic variables. In the real world, agents are often uncertain about the future effects of facts that they observe. Assuming that they are not aware of the exact nature of such facts is a simple and convenient way to model this kind of “conditional” uncertainty within the rational expectations paradigm.

---

<sup>1</sup>Beaudry and Lucke, 2009, and Dupaigne and Portier, 2006, find similar results.

Models with noisy information have important consequences for empirical analysis. In particular, standard VAR methods cannot be employed (Blanchard, L’Huillier and Lorenzoni, 2012, BLL henceforth). The reason is that economic variables, by reflecting agents’ behavior, can only convey information which is available to them. If agents cannot observe current structural shocks, current (and past) values of the economic time series cannot contain the relevant information to estimate such shocks. As a consequence, an econometrician will not be able to recover the structural shocks by a rotation of the VAR residuals. After all, if this were possible for the econometrician, it would be possible for the agents as well, contradicting the initial assumption.<sup>2</sup>

An equivalent formulation is that under imperfect information the structural shocks are *non-fundamental* with respect to agents’ information set (Hansen and Sargent, 1991, Lippi and Reichlin, 1993, 1994).<sup>3</sup>

This difficulty with the application of VAR methods is perhaps the main reason why the noisy-information approach has been seldom applied in this literature. Most empirical works about the business cycle effects of news, for instance, assume that news are noise free, i.e. that the structural shocks are observable (see e.g. Cochrane, 1994, Beaudry and Portier, 2006, Barsky and Sims, 2011, Forni, Gambetti and Sala, 2010). However If the information is corrupted by noise what is interpreted as the structural shock is actually a mixture of structural shock and noise.

By contrast, BLL and Barsky and Sims, 2012 (BS henceforth) assume noisy information and try to assess the role of noise (“animal spirits” in Barsky and Sims’ terminology) in driving output fluctuations. Both papers recognize that structural VARs are ill-suited and resort to direct estimation of the theoretical model. However, as convincingly argued by Sims, 1980, this approach requires strong *a priori* restrictions on the dynamic responses of the variables to the structural shocks. Such restrictions are necessarily arbitrary to a large extent, but may in principle have important effects on the final results. In

---

<sup>2</sup>Interesting and general results about the econometric implications of linear rational expectation models with incomplete information can be found in Baxter, Graham and Wright, 2011.

<sup>3</sup>This kind of non-fundamentality is different from the one that arises when the econometrician’s information set is narrower than that of the agents. In the latter case, the problem can be solved in principle by enlarging the data set (Forni, Giannone, Lippi and Reichlin, 2009, Forni and Gambetti, 2011).

fact, the two papers reach opposite conclusions: in BLL the noise has very large effects, whereas in BS “animal spirits” have essentially no effects.

In this paper we provide a non-standard structural VAR method, which allows estimation of the structural shocks and their effects under the assumption of imperfect observability. We begin the analysis by presenting a theoretical model in which agents observe the shock affecting future economic fundamentals, the real shock, *with noise*. The signal observed by the agents is the sum of the real shock and a “noise” shock. As time goes by, agents learn how much of the observed shock was noise and how much structural economic shock. In other words, future data perfectly reveal current structural shocks. This is the key mechanism which allows us to estimate the structural shocks. Indeed, while a contemporaneous linear combination of the VAR residuals cannot deliver the correct shock, a *dynamic* combination involving future residuals, can. More precisely, we show that, once the reduced form VAR has been estimated, the structural shocks and the corresponding impulse response functions can be obtained by applying suitable dynamic rotations (Blaschke transformations) to the residuals and the reduced-form impulse response functions.

Using this new approach we study the role of noise shocks as sources of business cycle fluctuations. We find that “noisy news”, the real and the noise shocks together, explain more than half of the fluctuations of GDP, consumption and investment. Expectations of future changes in economic fundamentals should be considered a major source of business cycle fluctuations. A large fraction of such fluctuations is due to noise shocks which generate hump-shaped responses of GDP, consumption and investment and account for about one third of their variance at short- and medium-run horizons. The role of noise is much larger than in BS, where “animal spirits” have negligible effects, and qualitatively different from BLL, where it is found to explain a very large fraction of consumption fluctuations on impact, but a relatively small fraction of consumption variance at the 3-year horizon and almost nothing of investment fluctuations.

The remainder of the paper is organized as follows. Section 2 discusses the economic model and the econometric implications; Section 3 presents the econometric model; Section 4 presents the empirical evidence; Section 5 concludes.

## 2 Some theory

In this section we present a simple model where agents decide the current level of consumption on the basis of expected future economic fundamentals. Economic fundamentals are driven by a *structural shock* which has delayed effects. Expectations are formed on the basis of a limited information set, in the sense that agents do not observe the current structural shock but only a noisy signal, the "noisy news". Precisely, agents observe the aggregate of the structural and the noise shock. The implication is that consumption reacts both to disturbances which actually affect future economic fundamentals and disturbances which do not have any effect.

### 2.1 A simple model

We assume that potential output,  $a_t$ , follows the exogenous relation

$$a_t = a_{t-1} + \varepsilon_{t-1}, \quad (1)$$

where the  $\varepsilon_t$  is a Gaussian, serially uncorrelated process affecting  $a_t$  with a one-period delay. We refer to this shock as "real" shock. Consumers observe a noisy signal of  $\varepsilon_t$ , the *signal* henceforth, given by

$$s_t = \varepsilon_t + v_t, \quad (2)$$

where the noise shock  $v_t$  is a Gaussian white noise, uncorrelated with  $\varepsilon_t$  at all leads and lags. The variance of the signal is just the sum of the variances of the two shocks,  $\sigma_s^2 = \sigma_\varepsilon^2 + \sigma_v^2$ . In addition, agents observe potential output  $a_t$ , so that the consumers' information set is given by present and past values of  $a_t$  and  $s_t$ , i.e.  $\mathcal{I}_t = \text{span}(a_{t-k}, s_{t-k}, k \geq 0)$ . Given the delayed effects of the real shock, this information is not sufficient to distinguish the current true real shock from noise. At time  $t + 1$ , however, consumers learn about the past realization of the two shocks since they observe  $\varepsilon_t = \Delta a_{t+1}$  and therefore  $v_t = s_t - \varepsilon_t$ .

Following BLLH, we assume that agents set consumption,  $c_t$ , on the basis of expected long-run fundamentals; precisely,

$$c_t = \lim_{j \rightarrow \infty} E(a_{t+j} | \mathcal{I}_t). \quad (3)$$

Realized output,  $y_t$ , is fully demand-determined, i.e.  $y_t = c_t$ ; employment adjusts to clear the labor market.

## 2.2 Solution and economic implications

Given the process for  $a_t$ , we have  $E(a_{t+j}|\mathcal{I}_t) = E(a_{t+1}|\mathcal{I}_t)$  for any  $j > 1$ , so that

$$c_t = E(a_{t+1}|\mathcal{I}_t) = a_t + E(\varepsilon_t|\mathcal{I}_t). \quad (4)$$

Since  $a_{t-k}$  and  $s_{t-k}$ , for  $k > 0$ , are uninformative about  $\varepsilon_t$ ,  $E(\varepsilon_t|\mathcal{I}_t)$  is simply the projection of  $\varepsilon_t$  on  $s_t$ , that is

$$E(\varepsilon_t|\mathcal{I}_t) = \gamma s_t$$

where  $\gamma = \sigma_\varepsilon^2/\sigma_s^2$ . Therefore  $c_t = a_t + \gamma(\varepsilon_t + v_t)$  and the change in consumption is

$$\begin{aligned} \Delta c_t &= \Delta a_t + \gamma \Delta(\varepsilon_t + v_t) \\ &= \gamma \varepsilon_t + (1 - \gamma)\varepsilon_{t-1} + \gamma v_t - \gamma v_{t-1}. \end{aligned} \quad (5)$$

Following a real shock, consumption immediately jumps by  $\gamma \varepsilon_t$  and in the second period reaches its new long run level  $c_{t-1} + \varepsilon_t$ . Consumption reacts also to the noise shock: following a positive noise shock, consumption increases by  $\gamma v_t$  on impact and then reverts back to its initial level  $c_{t-1}$  after one period. Notice that the impact responses are identical, since agents cannot distinguish between the two shocks in the current period. However, after one period, observed potential output unveils the nature of the shock and agents, recognizing it was noise, undo the initial increase by reducing consumption by  $\gamma v_t$ . While the real shock has a permanent effect, the noise shock has only a temporary effect.

It is instructive to compare these results with the case in which agents can observe the real shock without error. In this case, equation (4) implies  $c_t = a_t + \varepsilon_t$  and

$$\Delta c_t = \varepsilon_t,$$

so that after a real shock consumption jumps immediately to its new long run level.<sup>4</sup> Imperfect information has two implications. First, agents are more cautious in changing

---

<sup>4</sup>Notice that consumption is a random walk in both cases of complete and incomplete information. To see this, consider that the first order autocovariance of  $\Delta c_t$  in equation (5) is  $\sigma_\varepsilon^2 \gamma (1 - \gamma) - \sigma_v^2 \gamma^2 = \sigma_\varepsilon^2 \gamma - \gamma^2 (\sigma_\varepsilon^2 + \sigma_v^2) = \sigma_\varepsilon^2 \frac{\sigma_\varepsilon^2}{\sigma_s^2} - \frac{\sigma_\varepsilon^4}{\sigma_s^4} \sigma_s^2 = 0$ .

their consumption pattern. More precisely, for a given variance of the real shock, the higher is the variance of noise, the smaller is the contemporaneous change in consumption (recall that  $\gamma = \sigma_\varepsilon^2/\sigma_s^2$ ). Second, the noise shock can generate cyclical fluctuations in consumption and output completely unrelated to economic fundamentals.

Let us see how quantitatively important these fluctuations can be. The total variance of consumption change is  $\sigma_\varepsilon^2$ .<sup>5</sup> The contribution of the noise component is then

$$\frac{2\gamma^2\sigma_v^2}{\sigma_\varepsilon^2} = 2\frac{\sigma_\varepsilon^2}{\sigma_s^4}\sigma_v^2 = \frac{2\sigma_\varepsilon^2\sigma_v^2}{(\sigma_v^2 + \sigma_\varepsilon^2)^2},$$

which depends on the variance of the noise component. Let us consider the two limiting cases  $\sigma_v^2 = 0$  and  $\sigma_v^2 \rightarrow \infty$ . In the former case there is no noise, so that its contribution to total variance is obviously zero. In the latter case the signal is dominated by noise, so that it is not informative at all. Interestingly, the variance of the noise component approaches zero also in this case. The reason is that agents recognize that the signal is uninformative and do not react to it. Finally, it is easily seen that the above expression reaches its maximum when  $\sigma_v^2 = \sigma_\varepsilon^2$ . In this case 50% of the fluctuations of consumption change are due to noise.

### 2.3 The failure of standard structural VAR methods

Imperfect observability of structural shocks has important econometric implications. To see this, let us rewrite the solution of the model as

$$\begin{pmatrix} \Delta a_t \\ \Delta c_t \\ s_t \end{pmatrix} = \begin{pmatrix} L & 0 \\ \gamma + (1-\gamma)L & \gamma - \gamma L \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix}, \quad (6)$$

where  $L$  is the lag operator. To simplify things, let us further assume for the moment that the econometrician can observe  $s_t$ .

First, the econometrician (just like the agents) would not be able to recover real and noise shocks from the present and past values of  $a_t$  and  $s_t$ . It is easily seen from (6) that the polynomial matrix of the subsystem associated to  $\Delta a_t$  and  $s_t$  has determinant vanishing at zero, which implies that the corresponding bivariate MA representation

<sup>5</sup>From (5) we have  $\text{var}(\Delta c_t) = [\gamma^2 + (1-\gamma)^2]\sigma_\varepsilon^2 + 2\gamma^2\sigma_v^2 = 2\gamma^2\sigma_s^2 + (1-2\gamma)\sigma_\varepsilon^2 = 2\sigma_\varepsilon^4/\sigma_s^2 - 2\sigma_\varepsilon^4/\sigma_s^2 + \sigma_\varepsilon^2 = \sigma_\varepsilon^2$ .

is non-invertible and *non-fundamental*, so that a VAR representation in the structural shocks does not exist.

The econometrician could also use consumption, in addition to potential output and  $s_t$ , but still he/she would fail to recover the shock. For, the rank of the polynomial matrix in (6) is one for  $L = 0$ , which means that even this “tall” representation is non-invertible and non-fundamental. In other words, the two shocks cannot be obtained from present and past values of the three variables.

Non-fundamentalness is a debated issue in the structural VAR literature. Early references are Hansen and Sargent, 1991, and Lippi and Reichlin, 1993, 1994; more recent contributions include Giannone and Reichlin, 2006, Fernandez-Villaverde *et al.*, 2007, Chari *et al.*, 2008, Forni and Gambetti, 2011. Essentially, the problem is that standard SVAR methods assume that the structural shocks are linear combinations of the residuals obtained by estimating a VAR. If the structural MA representation of the variables included in the VAR is non-fundamental, the structural shocks are not linear combinations of such residuals, so that the method fails.<sup>6</sup>

In most of the economic literature, the structural shocks are elements of agents’ information set and non-fundamentalness may arise if the econometrician uses less information than the agents. In this case, non-fundamentalness can in principle be solved by enlarging the information set used by the econometrician (Forni, Giannone, Lippi and Reichlin, 2009, Forni and Gambetti, 2011). But in the present setting non-fundamentalness stems from agents’ ignorance and cannot be solved by adding variables to the VAR. The economic intuition is that agents’ behavior cannot reveal information that agents do not have. Consumption or other variables which are the outcome of agents’ decisions do not add anything to the information already contained in  $a_t$  and  $s_t$ . More generally, in mod-

---

<sup>6</sup>An MA representation is fundamental if and only if the associated matrix is full column rank (i.e. the rank is equal to the number of shocks) for all  $L$  with modulus less than one (see Rozanov, 1967, Ch. 2). This condition is slightly different from invertibility, since invertibility requires full column rank also for  $L$  with unit modulus. Hence non-fundamentalness implies non-invertibility, whereas the converse is not true. When the variables are cointegrated, for instance, the MA representation of the first differences is not invertible, but nonetheless can be fundamental. In such a case, non-invertibility can be easily circumvented by resorting to structural ECM or level VAR estimation. Non-fundamentalness is a kind of non-invertibility which cannot be solved in this way.



els assuming that agents cannot see the structural shocks, the structural representation is non fundamental for whatever set of observable variables. For, if it were, agents could infer the shocks from the variables themselves, contrary to the assumption.

## 2.4 Agents' innovations and structural shocks

As discussed above, the relevant shocks cannot be found by using standard VAR methods. Hence a question arises: what shocks would the econometrician recover by running a VAR for potential output and the signal? To answer to this question we need to find shocks which are fundamental for  $\Delta a_t$  and  $s_t$ . Starting with

$$\begin{pmatrix} \Delta a_t \\ s_t \end{pmatrix} = \begin{pmatrix} L & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix},$$

we easily get the representation

$$\begin{pmatrix} \Delta a_t \\ s_t \end{pmatrix} = \begin{pmatrix} 1 & L\sigma_\varepsilon^2/\sigma_s^2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix} \quad (7)$$

where

$$\begin{pmatrix} u_t \\ s_t \end{pmatrix} = \begin{pmatrix} L\frac{\sigma_v^2}{\sigma_s^2} & -L\frac{\sigma_\varepsilon^2}{\sigma_s^2} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix}. \quad (8)$$

Notice that  $u_t$  and  $s_t$  are jointly white noise and orthogonal with variance  $\sigma_u^2 = \sigma_v^2\sigma_\varepsilon^2/\sigma_s^2$  and  $\sigma_s^2$  respectively.<sup>7</sup> Moreover, the determinant of the matrix in (7) is 1, so that the MA representation (7) is fundamental, implying that  $u_t$  and  $s_t$  are innovations of the agents' information set. The shock  $u_t$  can be interpreted as the “learning” shock, as it represents the new information about past structural shocks, resulting from observing present and past  $\Delta a_t$  and  $s_t$ .

In conclusion, by running a VAR for  $\Delta a_t$  and  $s_t$ , the econometrician would not recover the structural shocks  $\varepsilon_t$  and  $v_t$ , but rather two shocks – learning and signal – which are combinations of present and *past* values of the structural shocks. Of course, standard identification schemes would fail, since no linear combination of the two innovations at time  $t$  can deliver the structural shocks.

<sup>7</sup>To see that  $u_t$  and  $v_t$  are jointly white noise, observe that the covariance of  $u_t$  and  $s_{t-1}$  is  $\sigma_v^2\sigma_\varepsilon^2/\sigma_s^2 - \sigma_\varepsilon^2\sigma_v^2/\sigma_s^2 = 0$ .

The next question is: can the true structural shocks be recovered and how? The answer is positive, provided that the future values of the fundamental shocks are used. As already observed, after one period the observation of potential output unveils the real or noise nature of the signal. Indeed, representation (8) can be inverted toward the future:

$$\begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix} = \begin{pmatrix} L^{-1} & \frac{\sigma_\varepsilon^2}{\sigma_s^2} \\ -L^{-1} & \frac{\sigma_v^2}{\sigma_s^2} \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix}. \quad (9)$$

The above equation shows that the structural shocks, though not recoverable as static linear combinations of the VAR residuals, can be obtained as *dynamic* linear combinations, involving future values. This is the key result we will use in the econometric section to identify real and noise shocks.

## 2.5 Agents’ “learning”: a comparison with BLLH and BS

A crucial novelty of our model with respect to existing literature is the agents’ learning process. For the sake of comparison, let us recast the BLLH model, with minor modifications, in our notation. BLLH assumes that  $a_t$  is the sum of two components: a permanent one (which may affect  $a_t$  on impact), driven by the shock  $\varepsilon_t$ , and a temporary one, driven by the shock  $\eta_t$ . More specifically,

$$a_t = a_{t-1} + (1 - \rho L)^{-1} \varepsilon_t + (1 - L)(1 - \rho L)^{-1} \eta_t. \quad (10)$$

The signal is the same as in our model and is given by equation (2). As in our model, agents can observe  $a_t$  and the signal  $s_t$ .

The key difference between this model and ours is the reason why observing  $a_t$  and  $s_t$  does not reveal the structural shocks. In our model, agents cannot see the structural shocks because the shock affecting potential output has delayed effects; in other words, because it is a real shock. On the other hand, in BLLH, non-observability is due to the fact that there is also a temporary shock; that is, there are three shocks and only two dynamically independent observable variables. Similarly, the model proposed in BS for productivity and the signal has three shocks and just two variables.

This has a crucial implication. In our model, as time goes by, agents can recover past shocks exactly: in the simple version of the model described above, they learn everything

after one period; in a more general setting (see section 3) agents learn gradually, but in the long run they can see past shocks without error. By contrast, in both BLLH and BS, agents never learn completely the real or noise nature of past shocks. In both models, the MA equilibrium representation for the observable variables is rectangular, with more columns than rows. For instance, in BLLH we have

$$\begin{pmatrix} \Delta a_t \\ s_t \end{pmatrix} = \begin{pmatrix} (1 - \rho L)^{-1} & 0 & (1 - L)(1 - \rho L)^{-1} \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ v_t \\ \eta_t \end{pmatrix}. \quad (11)$$

Obviously, (11) cannot be inverted, not even in the future: past shocks cannot be written as dynamic linear combinations of the observables.

Similarly, the implications of our model for VAR analysis are different from what is found in the previous literature. In the frameworks of BLLH and BS, VAR methods fail because it is impossible to estimate the impulse response functions of three independent shocks —as well as the shocks themselves— with a bivariate VAR. In our framework instead, as we will show below, SVAR models can be employed successfully, as long as dynamic identification is used.

### 3 The econometric model

In this section we generalize the simple model of section 2.4 and propose our dynamic identification procedure.

Dynamic structural VAR identification is discussed in detail in Lippi and Reichlin, 1994. In their more general framework, the conditions required to reach identification are very demanding. The econometrician should know the relevant unitary dynamic transformation (the so called “Blaschke matrix”), which is characterized by the roots of the determinant of the structural representation that are smaller than one in modulus. Economic theory can hardly provide such information.

In the present setting, however, a restriction arises quite naturally from the theory: the conceptual distinction between real and noise shocks requires that  $\Delta a_t$ , the variable representing economic fundamentals, *is not affected by noise at any lag*. As a consequence, the reaction of  $\Delta a_t$  to past signals  $s_{t-1}$ , and “true” real  $\varepsilon_{t-1}$ , are equal, up to

a multiplicative constant which is given by the signal-to-noise variance ratio. This in turn implies that the “wrong” roots of the structural representation are revealed by the impulse response function of  $\Delta a_t$  to the signal  $s_t$ , which can be estimated.

### 3.1 Structural and fundamental representations

Let us consider a more general specification for potential output,

$$\Delta a_t = c(L)\varepsilon_t, \quad (12)$$

where  $c(L)$  is a rational function in  $L$  with  $c(0) = 0$ . The structural representation becomes

$$\begin{pmatrix} \Delta a_t \\ s_t \end{pmatrix} = \begin{pmatrix} c(L) & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix}, \quad (13)$$

This representation is non-fundamental, since the determinant of the MA matrix,  $c(L)$ , vanishes by assumption for  $L = 0$ . This means that present and past values of the observed variables  $\Delta a_t$  and  $s_t$  contain strictly less information than present and past values of  $\varepsilon_t$  and  $v_t$ .

As we have seen above, stationarity of  $\Delta a_t$  and  $s_t$  entails that the two variables have a fundamental representation with orthogonal innovations. Such a representation can be found as follows. Let  $r_j$ ,  $j = 1, \dots, n$ , be the roots of  $c(L)$  which are smaller than one in modulus and

$$b(L) = \prod_{j=1}^n \frac{L - r_j}{1 - \bar{r}_j L}$$

where  $\bar{r}_j$  is the complex conjugate of  $r_j$ . Then let us consider the representation

$$\begin{pmatrix} \Delta a_t \\ s_t \end{pmatrix} = \begin{pmatrix} \frac{c(L)}{b(L)} & \frac{c(L)\sigma_\varepsilon^2}{\sigma_s^2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix}, \quad (14)$$

where

$$\begin{pmatrix} u_t \\ s_t \end{pmatrix} = \begin{pmatrix} b(L)\frac{\sigma_v^2}{\sigma_s^2} & -b(L)\frac{\sigma_\varepsilon^2}{\sigma_s^2} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix} \quad (15)$$

As before,  $u_t$  and  $s_t$  are orthogonal innovations for agents' information set, so that  $\mathcal{I}_t = \text{span}(u_{t-k}, s_{t-k}, k \geq 0)$ .<sup>8</sup>

---

<sup>8</sup>To see this, observe that the determinant of the matrix in (14), i.e.  $c(L)/b(L)$ , vanishes only for  $|L| \geq 1$  because of the very definition of  $b(L)$ .

The relation between the fundamental shocks and the structural shock is given by

$$\begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix} = \begin{pmatrix} b(F) & \frac{\sigma_v^2}{\sigma_s^2} \\ -b(F) & \frac{\sigma_v^2}{\sigma_s^2} \end{pmatrix} \begin{pmatrix} u_t \\ s_t \end{pmatrix}. \quad (16)$$

where  $F$  is the forward operator, i.e.  $F = L^{-1}$ .<sup>9</sup> As in the previous section, the structural shock depends on future fundamental innovations, with the difference that here the real and noise shocks contained in the signal get unveiled in the long run, rather than after one period.

We further assume that the signal  $s_t$  is not observed by the econometrician but there is one observable variable,  $z_t$ , which reveals the signal. In principle such a variable may depend on both  $s_t$  and  $u_t$ . Therefore we can write the representation of  $\Delta a_t$  and  $z_t$  as

$$\begin{pmatrix} \Delta a_t \\ z_t \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L) \end{pmatrix} \begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix} = \begin{pmatrix} \frac{c(L)\sigma_u}{b(L)} & \frac{c(L)\sigma_\varepsilon^2}{\sigma_s} \\ d(L)\sigma_u & f(L)\sigma_s \end{pmatrix} \begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix} \quad (17)$$

where, following the usual econometric convention, the shocks are normalized to have unit variance. Observe however that the above representation is not necessarily fundamental, since the determinant of the MA matrix depends on  $d(L)$  and  $f(L)$ . In order to have fundamentalness,  $z_t$  has to be sufficiently informative to reveal  $s_t$ . In the reminder of this section we assume fundamentalness of (17); in the empirical section we will test for this property.

Moreover,

$$\begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \end{pmatrix} = \begin{pmatrix} b(L)\frac{\sigma_v}{\sigma_s} & -b(L)\frac{\sigma_\varepsilon}{\sigma_s} \\ \frac{\sigma_\varepsilon}{\sigma_s} & \frac{\sigma_v}{\sigma_s} \end{pmatrix} \begin{pmatrix} \varepsilon_t/\sigma_\varepsilon \\ v_t/\sigma_v \end{pmatrix} \quad (18)$$

so that the structural representation is

$$\begin{pmatrix} \Delta a_t \\ z_t \end{pmatrix} = \begin{pmatrix} c(L)\sigma_\varepsilon & 0 \\ f(L)\sigma_\varepsilon + b(L)d(L)\frac{\sigma_\varepsilon\sigma_v^2}{\sigma_s^2} & f(L)\sigma_v - b(L)d(L)\frac{\sigma_v\sigma_\varepsilon^2}{\sigma_s^2} \end{pmatrix} \begin{pmatrix} \varepsilon_t/\sigma_\varepsilon \\ v_t/\sigma_v \end{pmatrix} \quad (19)$$

### 3.2 Dynamic identification

Dynamic identification of the structural shocks is done in two parts. First we estimate and identify the fundamental representation (17); second we identify (18). Given the estimates of the two representations, an estimate of representation (19) immediately follows.

---

<sup>9</sup>Observe that  $1/b(L) = b(F)$ .

More specifically, the steps are the following.

1. Estimate an unrestricted VAR for  $\Delta a_t$  and  $z_t$  and compute the MA representation.
2. Impose that  $a_{12}(0) = 0$ . This condition implies that  $s_t$  does not affect  $\Delta a_t$  and comes from the theoretical restriction  $c(0) = 0$ . In the bivariate case, this is sufficient to identify the two fundamental shocks  $u_t$  and  $s_t$  and obtain an estimate of all the elements of the matrix of the impulse response functions of representation (17).
3. Let us call  $\hat{a}_{12}(L)$  the estimate of  $c(L)\sigma_\varepsilon^2/\sigma_s$  (see equation (17)). An estimate  $\hat{b}(L)$  of  $b(L)$  can be obtained as follows. Compute the roots of  $\hat{a}_{12}(L)$  and select the roots which are smaller than one in modulus (of course, one out of these roots will be zero by construction, because of the identifying assumption  $c(0) = 0$  of step 1). Using the roots which are smaller than one in modulus, estimate the polynomial  $b(L)$  in equation (3.1).
4. Let  $\hat{a}_{11}(L)$  be the estimate of  $a_{11}(L)$ , i.e. our estimate of  $c(L)\sigma_u/b(L)$ , and observe that  $b(1) = 1$ . Estimate  $\sigma_\varepsilon/\sigma_v$  as the ratio<sup>10</sup>

$$\frac{\hat{a}_{12}(1)}{\hat{a}_{11}(1)}.$$

5. Using the property that:  $\sigma_v^2/\sigma_s^2 + \sigma_\varepsilon^2/\sigma_s^2 = 1$ ,  $\widehat{\sigma_\varepsilon/\sigma_s}$  and  $\widehat{\sigma_v/\sigma_s}$  are obtained as  $\sin(\arctan(\widehat{\sigma_\varepsilon/\sigma_v}))$  and  $\cos(\arctan(\widehat{\sigma_\varepsilon/\sigma_v}))$ , respectively.

These five steps give the estimates of all the elements of representations (17) and (18) and consequently of all the elements in (19).

The (normalized) structural shocks  $\varepsilon_t/\sigma_\varepsilon$  and  $v_t/\sigma_v$  can be estimated by inverting equation (18). Since the determinant of the matrix in (18)  $1/b(L) = b(F)$  involves future values of  $u_t$  and  $s_t$ , the structural shocks cannot be estimated consistently at the end of the sample. This is in line with the assumption that neither the agents, nor the econometrician can see the current values of the structural shocks. However, in the middle of the sample the future is known and (16) can in principle provide reliable estimates of  $\varepsilon_t/\sigma_\varepsilon$  and  $v_t/\sigma_v$ . Such estimates can be used in combination with the corresponding

---

<sup>10</sup>In practice we compute the cumulated long-run effects as the effects at forty quarters.

response functions to decompose the series into the real and noise components and assess their importance in terms of explained variance.

Let us remark that the theoretical restrictions appearing in the first line of representation (17) are only partially exploited for identification and therefore can be used for testing. Such restrictions entail that in the structural representation (19) the impulse response function of  $\Delta a_t$  to a noise shock be identically zero, an hypothesis that can be easily verified by looking at the confidence bands.<sup>11</sup>

### 3.3 Multivariate specifications

Let us now consider a multivariate extension of the bivariate model described so far. This model will be used in the empirical section to investigate the role of noisy news in generating cyclical fluctuations.

Let  $\Delta w_t$  be an  $n - 2$ -dimensional vector of additional variables. In order to have a square system, it is convenient to assume that there are also  $n - 2$  additional shocks, potentially affecting  $a_t$ . Equation (1) becomes

$$\Delta a_t = c(L)\varepsilon_t + g(L)e_t, \quad (20)$$

where  $e_t$  is an  $n - 2$ -dimensional white noise vector with identity variance covariance matrix, orthogonal to  $\varepsilon_t$  at all leads and lags, and  $g(L)$  is an  $n - 2$ -dimensional row vector of rational functions in  $L$ . Moreover, we assume for simplicity that agents can observe  $e_t$ .

Under these assumptions, the “innovation” representation can be written as

$$\begin{pmatrix} \Delta a_t \\ z_t \\ \Delta w_t \end{pmatrix} = \begin{pmatrix} \frac{c(L)\sigma_u}{b(L)} & \frac{c(L)\sigma_\varepsilon^2}{\sigma_s} & g(L) \\ d(L)\sigma_u & f(L)\sigma_s & p(L) \\ q(L) & h(L) & m(L) \end{pmatrix} \begin{pmatrix} u_t/\sigma_u \\ s_t/\sigma_s \\ e_t \end{pmatrix} \quad (21)$$

where  $p(L)$ ,  $q(L)$ ,  $h(L)$  and  $m(L)$  are conformable vectors and matrices of rational functions in  $L$ . Again, we assume fundamentality of such representation. The corresponding

---

<sup>11</sup>The identification restrictions 1-5 impose a zero impact effect and a zero long-run cumulated effect; but between lag 0 and the maximal lag the impulse response function can be significantly different from zero.

structural representation is obtained by postmultiplying the above matrix by

$$\begin{pmatrix} b(L)\frac{\sigma_v}{\sigma_s} & -b(L)\frac{\sigma_\varepsilon}{\sigma_s} & 0' \\ \frac{\sigma_\varepsilon}{\sigma_s} & \frac{\sigma_v}{\sigma_s} & 0' \\ 0 & 0 & I_{n-2} \end{pmatrix} \quad (22)$$

where 0 denotes the  $n - 2$ -dimensional column vector of zeros.

Within the multivariate framework, the condition that the real shock does not affect  $a_t$  on impact is no longer sufficient, alone, to identify the model. To identify the learning shock  $u_t$  and  $s_t$ , we impose a Cholesky triangularization with the ordering  $\Delta a_t$ ,  $z_t$  and  $\Delta w_t$ . The learning shock and  $s_t$  will be the first two Cholesky shocks. The reason for this identification is that we want to allow for a contemporaneous effect of  $u_t$  and  $s_t$  on  $\Delta w_t$ . The drawback of this identification scheme is that  $\Delta a_t$  is not allowed to react contemporaneously to  $s_t$  and  $e_t$ , while  $z_t$  is not allowed to react contemporaneously to  $e_t$ . For this reason we will also try a different scheme where  $\Delta w_t$  is ordered first,  $\Delta a_t$  and  $z_t$  second and third, respectively. The shocks  $u_t$  and  $s_t$  will be the last two Cholesky shocks.

## 4 Evidence

In this section, we apply the methods described above to study the role of real and noise as sources of business cycle fluctuations. The main conclusion is that both real and noise shocks explain a sizable fraction of the forecast error variance of GDP, consumption and investment at business cycle horizons.

### 4.1 Data

The first step of our empirical analysis is to choose two series for  $a_t$  and  $z_t$ . Remind that the former is the variable representing economic fundamentals, which is unaffected by noise, while the latter is a variable revealing the signal  $s_t$ .

To represent  $a_t$  we take the log of US potential GDP from the CBO (GDPPOT), divided by population aged 16 years or more (civilian noninstitutional population). We choose per-capita potential output rather than total factor productivity (TFP), which is widely used in the expectation-driven business cycle literature, because TFP does not



pass the test described at the end of Section 3.2., that is we find that it is significantly affected by noise, contrary to a basic assumption of our model.

We use expected business conditions within the next 12 months (E12M), which is a component of the consumer confidence index from the Michigan University Consumer Survey, to represent  $z_t$ .<sup>12</sup> In the robustness exercise below we try two alternative series for  $z_t$ , i.e. the Conference Board leading economic indicators index and the Standard & Poor’s index of 500 common stocks. The latter variable is obtained from the monthly S&P500 index provided by Datastream. We converted the series in quarterly figures by taking simple averages and divided the resulting series by the GDP implicit price deflator in order to express it in real terms. The resulting series is taken in logs.

Since we are interested in evaluating the business cycle effects of real and noise shocks, we take in addition from the NIPA tables real GDP, real consumption, obtained as the sum of nondurables and services, and real investment, obtained as the sum of private investment and durable consumption. All variables are divided by civilian noninstitutional population and taken in logs.

Finally, in order to test for fundamentalness of the VAR, expressed in representation (21), we use the principal components from a large data set of macroeconomic variables. Such variables, along with the corresponding transformations, are reported in the Appendix. The time span of all data is 1960 I — 2010 IV.

## 4.2 VAR specification and the fundamentalness test

Our benchmark VAR specification includes potential GDP, E12M, real GDP (GDP), real consumption of nondurables and services (CONS) and real private investment plus consumption of durables (INV). To avoid potential cointegration problems we estimate the VAR in levels. According to the AIC criterion we include four lags.

As explained in Section 3, identification is obtained by assuming that potential GDP reacts on impact only to the learning shock and that expected business conditions react on impact to the learning shock and the signal. This implies that the two shocks can

---

<sup>12</sup>Similar results, not shown here, are obtained with the expected confidence index and expected business condition over the next 5 years, which is another component of the consumer sentiment index, extensively discussed in BS.

be found as the first two Cholesky shocks of the model with potential GDP ordered first and expected business conditions ordered second. Such a scheme has the feature that GDP, consumption and investment can react on impact both to the learning shock and the signal  $s_t$ . The structural representation is obtained by following the procedure explained in Section 3. Before identifying shocks, impulse response functions from the VAR in levels have been differenced.

As a first step, we test for fundamentalness of representation (21) as suggested in Forni and Gambetti, 2011. The idea underlying their method is simple: if representation (21) is fundamental, i.e. if the variables used in the VAR span the information set of the agents, then the estimated shocks (learning and  $s_t$ ) must be orthogonal to all available past information. The same orthogonality necessary condition holds *a fortiori* for the structural shocks which are linear combination of present and future values of  $u_t$  and  $s_t$ .

To represent available macroeconomic information we take the principal components of the US macroeconomic data set reported in the Appendix. Table 1 reports the p-values of the F-test of the regression of the estimated shocks on 2 and 4 lags of the first  $j$  principal components, with  $j = 1, \dots, 6$ . The null of orthogonality is never rejected.

For comparison, we report the corresponding results for the VAR including only GDPPOT and E12M (Table 2). For the bivariate specification, orthogonality of  $s_t$ , real and noise shocks is rejected, indicating that potential income and the confidence index do not convey enough information to recover the signal  $s_t$  and the structural shocks.

### 4.3 Impulse response functions

Figures 1 and 2 depict the impulse response functions of the five variables to learning and signal shocks. Shaded areas represent confidence bands at the 90% level constructed using the Kilian (1998)'s method. As expected, the signal shock has a large and significant impact effect on consumers' confidence and anticipates significantly future potential GDP. Moreover, it has a positive and significant impact effect on consumption, investment and realized GDP, reaching its maximum at the 2-year horizon. Afterwards, the effect declines, while, at the same time, the effect of learning increases and becomes significant. As agents learn about the past real and noise shocks by looking at potential GDP, they partially correct their previous response to the signal.

Figure 3 reports the impulse response functions of potential GDP and E12M to real and noise shocks. The noise, as predicted by the model, has no effects on potential output at all horizons. On the contrary, the response of potential output to the real shock increases steadily, after a zero initial effect, reaching its new long run level after about five years.

As for the consumer confidence indicator, both real and noise shocks have a significant impact effect, but the effect of noise is larger, reflecting the estimate of  $\sigma_\varepsilon/\sigma_s$  which is only 0.40, as against an implied estimate of  $\sigma_v/\sigma_s$  of 0.91.

Next we turn our attention to GDP, consumption and investment (Figure 4). The responses of the three variables to all shock have similar shapes. In the case of the noise shock, the responses are hump-shaped with a relatively small, although significant, impact effect; they reach a maximum after about two years, then decline approaching zero after about five years. On the contrary, the responses to genuine real shocks are permanent. As predicted by the model, noise shocks spur a wave of private consumption and investment which vanishes once economic agents realize that the signal was just noise.

#### 4.4 Variance decomposition

Variance decompositions are reported in Table 3. The signal shock explains a relatively small fraction of potential output volatility (about 23% at the four-year horizon), but a very large fraction of realized GDP, consumption and investment (about 50-60% at the 2-year and the 4-year horizons). This seems consistent with the general idea that signals, while providing a rather imperfect anticipation of future changes of economic fundamentals, are an important source of business cycle fluctuations.

Turning to the analysis of real and noise shocks, business cycle fluctuations are largely driven by noise, which accounts for 30-40% of the forecast error variance of the three variables at the two-year horizon. The real shock has a sizable, but more limited role in the short run, accounting for about 20-25% of the variance of GDP and consumption at the 2-year horizon. Investment, in particular, is largely dominated by noise, which explains 45% of fluctuations at the 2-year horizon, as against only 5% for “genuine” real shocks.

Real shocks explain most of the variance of potential output at all horizons. By contrast, consistently with the impulse response functions of Figure 3, the variance of the consumer confidence indicator E12M is largely dominated by noise shocks. In fact, we have seen that E12M anticipates a good deal of transitory fluctuations in GDP, unrelated to potential GDP. This findings supports the “animal spirit” interpretation of consumer sentiment, in contrast with BS, where fluctuations of confidence indicators are almost entirely attributable to real shocks.

Noise and real shock together explain more than half of the fluctuations GDP, consumption and investment at horizons ranging from 2 to 4 years. This finding and the fact that the two shocks generate positive co-movements between GDP, consumption and investment in the short and medium run, drives us to the main conclusion that noisy expectations of future changes in economic fundamentals, which in large part do not eventually materialize, should be considered a major source of business cycles.

Let us discuss the case of an econometrician that assumes that noise shocks do not exist. He/she therefore mistakenly identifies the signal shock as the real shock (when real shocks are absent, a shock that does not move on impact potential output is a real shock) and will conclude that real shocks explain approximately 50-60% of real variables. The econometrician will therefore attribute a higher role to real shocks, while a significant part of business cycle fluctuations is driven by noise shocks.

The results on the relative role of real and noise shocks differ substantially with what found in previous literature. First, the role of noise is much larger than in BS, where “animal spirits” have negligible effects. Second, they are qualitatively different from what found in BLLH, where noise explain a very large fraction of consumption fluctuations on impact, a small fraction of consumption variance at the 3-year horizon, and almost nothing of investment fluctuations at all horizons. Such large differences call for some explanations. For the reasons explained in Section 2, the results of BLLH and BS are not obtained by estimating a structural VAR. They specify a theoretical model and estimate the parameters of the model. A shortcoming of such procedure is that it requires strong *a priori* restrictions on the dynamic responses of the variables to the structural shocks. For instance, BLLH assumes that the impulse response function of  $\Delta a_t$  to the real shock is  $1/(1-\rho L)$ , whereas BS assumes  $L/(1-\alpha L)$ . Both models assume

that there is a second shock affecting productivity; BS assumes a permanent shock with no dynamic at all, whereas BLLH assumes a transitory shock with response function  $(1 - L)/(1 - \rho L)$ , the parameter  $\rho$  being the same as before. Clearly such restrictions are arbitrary to a large extent and may in principle have important effects on the final results. From this point of view, structural VAR methods have the advantage that the dynamic shape of the impulse response functions is quite general. Here, impulse response functions are obtained by imposing standard Choleski identification restrictions, along with the condition that  $\Delta a_t$  does not react contemporaneously to noise.

#### 4.5 Historical decomposition

Figure 5 reports the yearly growth rates of GDP (top panel) and the cyclical component of real GDP (bottom panel), as well as the component of the two series due to the noise shock over the last two decades.<sup>13</sup>

Several interesting results emerge. First, during the boom of the late 90s the noise is responsible for about half of the growth rate of GDP. Second, the shock substantially contributes to the 2001 recession and the slow recovery of the following two years. The low pace consumption and investment growth of the 2002 and 2003, according to the picture, was largely attributable to bad signals about future potential output outcomes which ex-post turned out to be just noise. Between 2004 and 2006 the shock again substantially contributed to the economic expansion. It is interesting to notice that the periods 1995-2000 and 2003-2006 were associated to asset prices bubbles.

In a companion paper, we show that noise in stock prices fully explains the information technology boom of the stock market at the end of the nineties and the subsequent burst (Forni, Gambetti, Lippi and Sala, 2013).

#### 4.6 Alternative identifications

The drawback of our identification procedure is that, presumably, also other shocks in addition to learning and the signal, could affect expected economic conditions contemporaneously. For this reason we implement two alternative identifications. In the

---

<sup>13</sup>The cyclical component of GDP is obtained by filtering the log of per-capita GDP with a band-pass filter retaining waves of periodicity between 6 and 32 quarters.

first, potential GDP and expected business conditions are ordered fourth and fifth, respectively, after GDP, consumption and investment. Within this identification, both potential output and consumer sentiment are allowed to react contemporaneously to all other shocks.

Figure 6 and 7 reports the impulse response functions for the first alternative identification (dashed lines) as well as the point estimates and the confidence bands obtained in the benchmark specification (solid line and gray areas). The results of the two identification schemes are qualitatively and quantitatively similar. Table 4 reports the variance decomposition. The fraction of forecast error variance attributable to both noise and real shocks is slightly reduced as compared with Table 3, but the two shocks taken together still account for about 50-60% of the variance of the three variables at the 4-year horizon.

In the second alternative identification, the federal funds rate is ordered second after potential GDP and before consumer sentiment, GDP, consumption and investment. This check is important to rule out the possibility that the noise shock actually captures monetary policy shocks which also should have no effects on potential output. In general the results (available upon request) are very similar to those obtained with the baseline specification. For instance at a 2-year horizon the noise shocks explains 39%, 33% and 45% of GDP, consumption and investment forecast error variance respectively as against 40%, 32% and 45% of Table 3.

#### **4.7 Alternative proxies for the signal**

In this subsection we repeat the analysis done in the previous section for the benchmark specification using different proxies for the signal. In particular we replace expected business conditions with real stock prices (S&P500 deflated by the DGP deflator) and the Conference Board Leading Economic Indicators Index. The two variables are ordered second after the potential GDP. Figure 8 plots the impulse response obtained in the two new specifications (dashed and dashed-dotted lines) as well as the point estimate and the confidence bands obtained in the benchmark specification (solid line and gray areas).

The results for the new specifications are again qualitatively similar to those obtained in the benchmark case with a few differences. In particular, with stock prices the responses of GDP, consumption and investment to the noise shock tend to be larger than

those obtained with the two other specifications, and, consequently noise shocks even more important for cyclical fluctuations.

Figures 9 and 10 report the historical decomposition of GDP for the two new specifications. The results are similar to those obtained in the benchmark specification. The ups and downs in the noise component are well synchronized with fluctuations in GDP. The noise shock substantially contributes to the two booms and the 2001 recession.

## 5 Conclusions

In this paper we have presented a business cycle model where agents receive imperfect signals about future economic fundamentals. We have shown that in this model the structural MA representation of economic variables is non-fundamental, so that standard structural VAR methods fail. We have argued that this is a general feature of models where economic agents cannot see the structural shocks.

As times goes by, both the agents and the econometrician learn about past structural shocks. A distinguishing feature of our model is that the structural shocks can be recovered exactly from future information. This is because, unlike existing models with imperfect information, the number of structural shocks is equal to the number of independent sources of informations observed by the agents. We have shown that in this case structural VARs can still be successfully used to estimate the structural shocks and the related impulse response functions, provided that identification is generalized to include dynamic transformations of VAR residuals.

In the empirical section, we have estimated a VAR and imposed a dynamic scheme to identify real and noise shocks and the related impulse response functions. We have found that noise and real shocks together explain more than half of the fluctuations of GDP, consumption and investment. A large fraction of such fluctuations is due to noise shocks which generate hump-shaped responses of GDP, consumption and investment and account for about one third of their variance at short- and medium-run horizons. The role of noise shocks is much larger than in BS, where “animal spirits” have negligible effects, and qualitatively different from BLLH, where it explains a very large fraction of consumption fluctuations on impact, but a relatively small fraction of consumption variance at the 3-year horizon and almost nothing of investment fluctuations.

## Appendix: Data (For Online Publication)

Transformations: 1 = levels, 2 = logs, 3 = first differences of logs. Most series are taken from the FRED database. TFP data are taken from the Federal Reserve Bank of San Francisco database. A few stock market and leading indicators are taken from Datastream. Monthly data have been temporally aggregated to get quarterly figures. CNP = Civilian Noninstitutional Population (Fred mnemonic: CNP16OV).



no.series	Transf.	Mnemonic	Long Label
1	2	GDPC1/CNP	Real Gross Domestic Product/CNP
2	2	GNPC96/CNP	Real Gross National Product/CNP
3	2	(NICUR/GDPDEF)/CNP	(National Income/GDP Deflator)/CNP
4	2	DPIC96/CNP	Real Disposable Personal Income/CNP
5	2	OUTNFB/CNP	Nonfarm Business Sector: Output/CNP
6	2	FINSLC1/CNP	Real Final Sales of Domestic Product/CNP
7	2	(FPIC1+PCNDGC96)/CNP	(Real Private Fixed Inv. + Real Durables Cons.)/CNP
8	2	PRFIC1/CNP	Real Private Residential Fixed Investment/CNP
9	2	PNFIC1/CNP	Real Private Nonresidential Fixed Investment/CNP
10	2	GPDIC1/CNP	Real Gross Private Domestic Investment/CNP
11	2	(PCNDGC96+PCESVC96)/CNP	(Real Pers. Cons. Exp.: Non Durables + Services)/CNP
12	2	PCNDGC96/CNP	Real Pers. Cons. Exp.: Nondurable Goods /CNP
13	2	PCDGC96/CNP	Real Pers. Cons. Exp.: Durable Goods/CNP
14	2	PCESVC96/CNP	Real Pers. Cons. Exp.: Services/CNP
15	2	(GSAVE/GDPDEF)/CNP	(Gross Saving/GDP Deflator)/CNP
16	2	FGCEC1/CNP	Real Federal Cons. Exp. & Gross Investment/CNP
17	2	(FGEXPND/GDPDEF)/CNP	(Federal Gov.: Current Exp./ GDP Deflator)/CNP
18	2	(FGRECPT/GDPDEF)/CNP	(Federal Gov. Current Receipts/ GDP Deflator)/CNP
19	1	CBIC1	Real Change in Private Inventories
20	2	EXPGSC1/CNP	Real Exports of Goods & Services /CNP
21	2	IMPGSC1/CNP	Real Imports of Goods & Services /CNP
22	2	CP/GDPDEF	Corporate Profits After Tax/GDP Deflator
23	2	NFCPATAX/GDPDEF	Nonfin. Corp. Bus.: Profits After Tax/GDP Deflator
24	2	CNCF/GDPDEF	Corporate Net Cash Flow/GDP Deflator
25	2	DIVIDEND/GDPDEF	Net Corporate Dividends/GDP Deflator
26	2	HOANBS/CNP	Nonfarm Business Sector: Hours of All Persons/CNP
27	2	OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons
28	2	UNLPNBS	Nonfarm Business Sector: Unit Nonlabor Payments
29	2	ULCNFB	Nonfarm Business Sector: Unit Labor Cost
30	2	WASCUR/CPI	Compensation of Employees: Wages & Salary Accruals/CPI
31	3	COMPNFB	Nonfarm Business Sector: Compensation Per Hour
32	2	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour
33	3	GDPCTPI	Gross Domestic Product: Chain-type Price Index
34	3	GNPCTPI	Gross National Product: Chain-type Price Index
35	3	GDPDEF	Gross Domestic Product: Implicit Price Deflator
36	3	GNPDEF	Gross National Product: Implicit Price Deflator
37	2	INDPRO	Industrial Production Index
38	2	IPBUSEQ	Industrial Production: Business Equipment
39	2	IPCONGD	Industrial Production: Consumer Goods

no.series	Transf.	Mnemonic	Long Label
40	2	IPDCONGD	Industrial Production: Durable Consumer Goods
41	2	IPFINAL	Industrial Production: Final Products (Market Group)
42	2	IPMAT	Industrial Production: Materials
43	2	IPNCONGD	Industrial Production: Nondurable Consumer Goods
44	1	AWHMAN	Average Weekly Hours: Manufacturing
45	1	AWOTMAN	Average Weekly Hours: Overtime: Manufacturing
46	1	CIVPART	Civilian Participation Rate
47	2	CLF16OV	Civilian Labor Force
48	2	CE16OV	Civilian Employment
49	2	USPRIV	All Employees: Total Private Industries
50	2	USGOOD	All Employees: Goods-Producing Industries
51	2	SRVPRD	All Employees: Service-Providing Industries
52	2	UNEMPLOY	Unemployed
53	2	UEMPMEAN	Average (Mean) Duration of Unemployment
54	1	UNRATE	Civilian Unemployment Rate
55	2	HOUST	Housing Starts: Total: New Privately Owned Housing Units Started
56	1	FEDFUNDS	Effective Federal Funds Rate
57	1	TB3MS	3-Month Treasury Bill: Secondary Market Rate
58	1	GS1	1- Year Treasury Constant Maturity Rate
59	1	GS10	10-Year Treasury Constant Maturity Rate
60	1	AAA	Moody's Seasoned Aaa Corporate Bond Yield
61	1	BAA	Moody's Seasoned Baa Corporate Bond Yield
62	1	MPRIME	Bank Prime Loan Rate
63	3	M1SL	M1 Money Stock
64	3	M2MSL	M2 Minus
65	3	M2SL	M2 Money Stock
66	3	BUSLOANS	Commercial and Industrial Loans at All Commercial Banks
67	3	CONSUMER	Consumer (Individual) Loans at All Commercial Banks
68	3	LOANINV	Total Loans and Investments at All Commercial Banks
69	3	REALLN	Real Estate Loans at All Commercial Banks
70	3	TOTALSL	Total Consumer Credit Outstanding
71	3	CPIAUCSL	Consumer Price Index For All Urban Consumers: All Items
72	3	CPIULFSL	Consumer Price Index for All Urban Consumers: All Items Less Food
73	3	CPILEGSL	Consumer Price Index for All Urban Consumers: All Items Less Energy
74	3	CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy
75	3	CPIENGSL	Consumer Price Index for All Urban Consumers: Energy
76	3	CPIUFDSL	Consumer Price Index for All Urban Consumers: Food
77	3	PPICPE	Producer Price Index Finished Goods: Capital Equipment
78	3	PPICRM	Producer Price Index: Crude Materials for Further Processing
79	3	PPIFCG	Producer Price Index: Finished Consumer Goods
80	3	PPIFGS	Producer Price Index: Finished Goods

no.series	Transf.	Mnemonic	Long Label
81	3	OILPRICE	Spot Oil Price: West Texas Intermediate
82	3	USSHRPRCF	US Dow Jones Industrials Share Price Index (EP) NADJ
83	2	US500STK	US Standard & Poor's Index if 500 Common Stocks
84	2	USI62...F	US Share Price Index NADJ
85	2	USNOIDN.D	US Manufacturers New Orders for Non Defense Capital Goods (B CI 27)
86	2	USCNORCGD	US New Orders of Consumer Goods & Materials (BCI 8) CONA
87	1	USNAPMNO	US ISM Manufacturers Survey: New Orders Index SADJ
88	2	USCYLEAD	US The Conference Board Leading Economic Indicators Index S ADJ
89	2	USECRIWLH	US Economic Cycle Research Institute Weekly Leading Index
90	2	GEXPND/GDPDEF	(Government Current Expenditures/ GDP Deflator)
91	2	GRECPT/GDPDEF	(Government Current Receipts/ GDP Deflator)
92	2	GCEC1	Real Government Consumption Expenditures & Gross Investment
93	2		Fernald's TFP growth CU adjusted
94	2		Fernald's TFP growth
95	2		(DOW JONES/GDP Deflator)/Civilian Noninstitutional Population
96	2		(S&P500/GDP Deflator)/Civilian Noninstitutional Population
97	2		Fernald's TFP growth - Investment
98	2		Fernald's TFP growth - Consumption
99	2		Fernald's TFP growth CU - Investment
100	2		Fernald's TFP growth CU - Consumption
101	1		Michigan Consumer Sentiment: Personal Finance Current
102	1		Michigan Consumer Sentiment: Personal Finance Expected
103	1		Michigan Consumer Sentiment: Business Condition 12 Months
104	1		Michigan Consumer Sentiment: Business Condition 5 Years
105	1		Michigan Consumer Sentiment: Buying Conditions
106	1		Michigan Consumer Sentiment: Current Index
107	1		Michigan Consumer Sentiment: Expected Index
108	2	GDPPOT	US Potential Output from the CBO

## References

- [1] Angeletos G., M. and J. La'O, (2010). Noisy Business Cycles, NBER Chapters, in: NBER Macroeconomics Annual 24, 319-378.
- [2] Barsky, R. and E. Sims (2012). Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence, *American Economic Review* 102, 1343-77.
- [3] Barsky, R. and E. Sims (2011). News shocks and business cycles, *Journal of Monetary Economics* 58, 273-289.
- [4] Baxter, B., Graham, L. and Wright, S. (2011). "Invertible and non-invertible information sets in linear rational expectations models," *Journal of Economic Dynamics and Control* 35, pp. 295-311.
- [5] Beaudry, P. and F. Portier (2004). Exploring Pigou's Theory of Cycles, *Journal of Monetary Economics* 51, 1183-1216.
- [6] Beaudry, P. and F. Portier (2006). Stock Prices, News, and Economic Fluctuations. *American Economic Review* 96, 1293-1307.
- [7] Blanchard O.J., G. Lorenzoni and J.P L'Huillier (2010). News, Noise, and Fluctuations: An Empirical Exploration. NBER Working Papers 15015.
- [8] Chari, V.V., Kehoe, P.J. and E.R. McGrattan (2008). Are structural VARs with long-run restrictions useful in developing business cycle theory? *Journal of Monetary Economics* 55, 1337-1352.
- [9] Christiano, L., C. Ilut, R. Motto and M. Rostagno (2007). Signals: Implications for Business Cycles and Monetary Policy, mimeo, Northwestern University.
- [10] Cochrane, John H. (1994). Shocks, *Carnegie-Rochester Conference Series on Public Policy* 41, 295-364.
- [11] Den Haan, W.J. and G. Kaltenbrunner (2009). "Anticipated growth and business cycles in matching models," *Journal of Monetary Economics* 56, 309-327

- [12] Hansen, L.P., and T.J. Sargent (1991). Two problems in interpreting vector autoregressions. In *Rational Expectations Econometrics*, L.P. Hansen and T.J. Sargent, eds. Boulder: Westview, pp.77-119.
- [13] Fernandez-Villaverde, J., J. F. Rubio, T. Sargent and M. Watson (2007). A, B, C, (and D)'s for Understanding VARs, *American Economic Review* 97, 1021-1026.
- [14] Forni, M., L. Gambetti (2010b). Fiscal Foresight and the Effects of Government Spending, CEPR Discussion Paper Series no. 7840.
- [15] Forni, M. and Gambetti, L., 2011, "Sufficient information in structural VARs," Center for Economic Research (RECent) 062, University of Modena and Reggio Emilia, Dept. of Economics.
- [16] Forni, M., L. Gambetti and L. Sala (forthcoming). No News in Business Cycles. *The Economic Journal*.
- [17] Forni, M., D. Giannone, M. Lippi and L. Reichlin (2009). Opening the Black Box: Structural Factor Models with Large Cross-Sections, *Econometric Theory* 25, 1319-1347.
- [18] Giannone, D., and L. Reichlin (2006). Does Information Help Recovering Structural Shocks from Past Observations? *Journal of the European Economic Association* 4, 455-465.
- [19] Hansen, L.P., and T.J. Sargent (1991). Two problems in interpreting vector autoregressions. In *Rational Expectations Econometrics*, L.P. Hansen and T.J. Sargent, eds. Boulder: Westview, pp.77-119.
- [20] Jaimovich, N. and S. Rebelo, (2006). Can News about the Future Drive the Business Cycle? mimeo, Northwestern University.
- [21] Keynes, J.M., 1936, *The general theory of employment, interest and money*. London: Macmillan.
- [22] Kilian, L. (1998). Small-Sample Confidence Intervals for Impulse Response Functions. *Review of Economics and Statistics* 80, 21830.

- [23] Leeper, E.M., Walker, T.B. and S.S. Yang (2011). Foresight and Information Flows. NBER Working Papers 16951.
- [24] Lippi, M. and L. Reichlin (1993). The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment, *American Economic Review* 83, 644-652.
- [25] Lippi, M. and L. Reichlin (1994). VAR analysis, non fundamental representation, Blaschke matrices, *Journal of Econometrics* 63, 307-325.
- [26] Lorenzoni, G., (2009). A Theory of Demand Shocks, *American Economic Review*, 99, 2050-84.
- [27] Lorenzoni, G. (2010). "Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information, *Review of Economic Studies* 77, pp. 305-338.
- [28] Lucas, R.E., Jr. (1972). Expectations and the Neutrality of Money. *Journal of Economic Theory* 4, 103-124.
- [29] Mankiw G. N. and R Reis (2002). Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *Quarterly Journal of Economics* 117, 1295-1328.
- [30] Pigou, A. C. (1927). "Industrial fluctuations", London, Macmillan.
- [31] Rozanov, Yu. (1967). *Stationary Random processes*. San Francisco: Holden Day.
- [32] Schmitt-Grohe, S. and M. Uribe, (2008). What's News in Business Cycles, NBER Working Papers 14215.
- [33] Sims, C. A. (2003). Implications of Rational Inattention. *Journal of Monetary Economics* 50, 665-690.
- [34] Woodford M. (2002). Imperfect Common Knowledge and the Effects of Monetary Policy, in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton: Princeton University Press.

Shock	Lags	Principal Components					
		1	2	3	4	5	6
Learning	2	0.82	0.83	0.48	0.70	0.72	0.68
	4	0.88	0.93	0.66	0.80	0.87	0.68
Signal	2	0.85	0.55	0.65	0.54	0.63	0.79
	4	0.97	0.83	0.84	0.78	0.82	0.93
Real	2	0.95	0.75	0.92	0.85	0.73	0.61
	4	0.95	0.91	0.66	0.74	0.78	0.73
Noise	2	0.60	0.40	0.42	0.51	0.65	0.78
	4	0.79	0.67	0.71	0.79	0.86	0.91

Table 1: Results of the fundamentalness test in the 5-variable VAR. Each entry of the table reports the  $p$ -value of the  $F$ -test in a regression of the shock on 2 and 4 lags of the first differences of the first  $j$  principal components,  $j = 1, \dots, 6$ .

Shock	Lags	Principal Components					
		1	2	3	4	5	6
Learning	2	0.97	0.80	0.60	0.79	0.70	0.75
	4	0.87	0.80	0.66	0.75	0.70	0.49
Signal	2	0.09	0.02	0.02	0.00	0.00	0.01
	4	0.24	0.06	0.03	0.02	0.03	0.10
Real	2	0.32	0.22	0.29	0.26	0.20	0.21
	4	0.43	0.45	0.13	0.16	0.26	0.34
Noise	2	0.17	0.05	0.03	0.02	0.02	0.04
	4	0.31	0.12	0.10	0.11	0.11	0.14

Table 2: Results of the fundamentalness test in the bivariate VAR. Each entry of the table reports the  $p$ -value of the  $F$ -test in a regression of the shock on 2 and 4 lags of the first differences of the first  $j$  principal components,  $j = 1, \dots, 6$ .



Variable	Horizon				
	Impact	1-Year	2-Years	4-Years	10-Years
	Learning				
GDPPOT	100.0	91.5	78.8	62.6	49.6
E12M	0.1	1.5	5.0	8.1	8.8
GDP	6.4	3.4	4.7	16.3	29.0
CONS	15.3	6.5	7.0	17.7	32.3
INV	0.5	0.9	0.7	4.0	12.3
	Signal				
GDPPOT	0.0	2.9	12.5	23.0	15.2
E12M	99.9	88.9	79.4	65.6	59.4
GDP	8.2	37.2	58.1	57.2	31.7
CONS	5.5	32.3	50.2	54.3	30.0
INV	7.7	36.1	49.0	47.5	35.3
	Real				
GDPPOT	0.0	87.4	87.6	81.4	63.1
E12M	16.3	15.1	21.2	23.3	21.2
GDP	1.4	15.7	22.2	39.3	44.3
CONS	1.0	18.7	23.9	41.6	48.9
INV	1.3	3.3	4.6	11.8	18.5
	Noise				
GDPPOT	0.0	4.6	2.1	3.1	1.1
E12M	83.7	75.1	63.0	51.1	47.1
GDP	7.3	24.6	40.0	33.5	16.2
CONS	5.5	19.3	32.4	29.6	13.1
INV	6.5	33.5	45.0	39.3	29.0
	Real+Noise				
GDPPOT	0.0	92.0	89.7	84.5	64.1
E12M	100.0	90.2	84.2	74.4	68.3
GDP	8.7	40.2	62.2	72.8	60.4
CONS	6.5	38.0	56.3	71.2	62.0
INV	7.8	36.8	49.7	51.1	47.5

Table 3: Variance decomposition in the 5-variable VAR, E12M ordered second. The entries are the percentage of variance explained by the shocks.

Variable	Horizon				
	Impact	1-Year	2-Years	4-Years	10-Years
	Learning				
GDPPOT	74.9	70.3	58.8	49.1	45.2
E12M	1.2	2.6	6.5	12.3	12.5
GDP	0.0	0.9	1.0	11.5	26.8
CONS	0.0	1.1	1.0	10.2	28.3
INV	0.0	1.9	1.3	7.9	20.7
	Signal				
GDPPOT	0.0	3.2	13.1	26.3	21.0
E12M	88.2	83.6	81.2	69.5	62.6
GDP	0.0	13.0	36.1	48.9	31.1
CONS	0.0	14.1	33.2	48.1	31.5
INV	0.0	15.2	33.1	42.8	34.0
	Real				
GDPPOT	0.0	61.6	66.7	71.4	64.7
E12M	21.9	16.8	27.0	32.7	30.1
GDP	0.0	1.3	7.4	31.4	43.9
CONS	0.0	1.2	6.9	30.0	47.1
INV	0.0	1.2	4.5	20.9	32.9
	Noise				
GDPPOT	0.0	5.8	2.2	2.8	1.0
E12M	67.4	69.3	60.5	49.0	45.0
GDP	0.0	12.4	29.4	28.2	13.7
CONS	0.0	13.8	27.1	27.5	12.3
INV	0.0	15.3	29.9	29.1	21.6
	Real+Noise				
GDPPOT	0.0	67.4	72.9	76.4	65.1
E12M	89.3	86.1	87.1	80.3	73.3
GDP	0.0	13.7	36.8	59.6	57.6
CONS	0.0	15.0	34.0	57.5	59.4
INV	0.0	16.5	34.4	50.0	54.5

Table 4: Variance decomposition in the 5-variable VAR, E12M ordered last. The entries are the percentage of variance explained by the shocks.

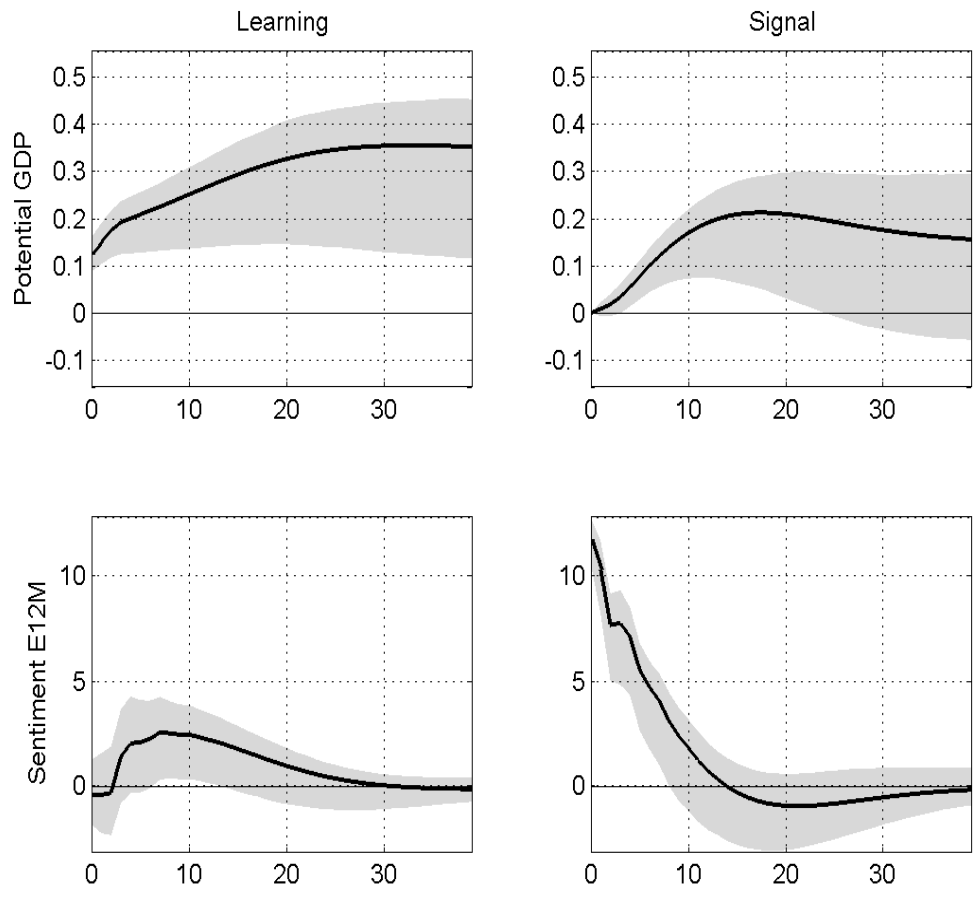


Figure 1: Impulse response functions to learning (left column) and signal (right column) shocks in the 5-variable VAR. Solid line: point estimate. Grey area: 90% confidence bands.

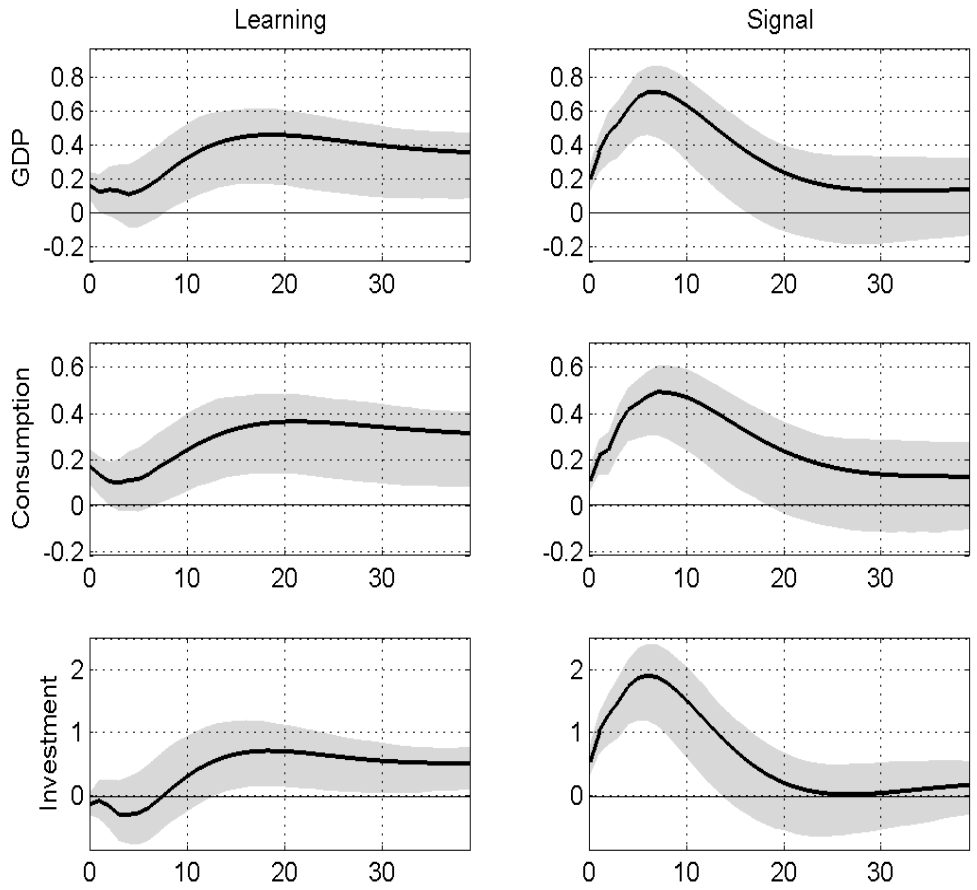


Figure 2: Impulse response functions to learning (left column) and signal (right column) shocks in the 5-variables VAR. Solid line: point estimate. Grey area: 90% confidence bands.

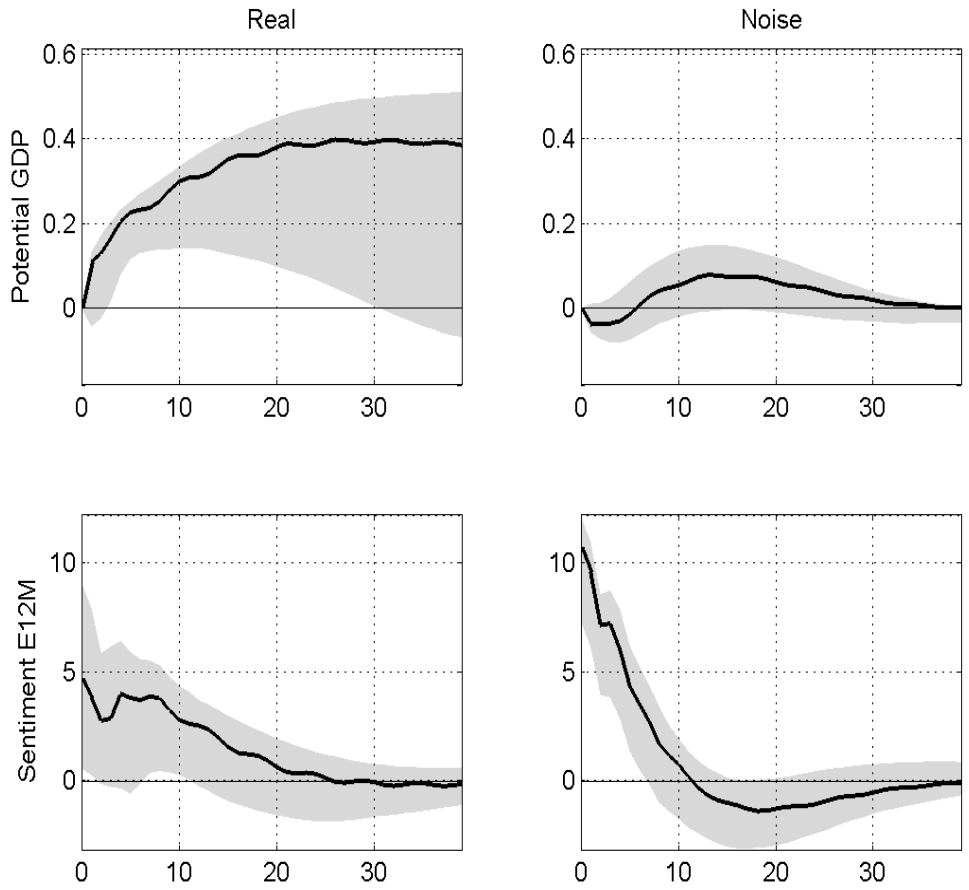


Figure 3: Impulse response functions to real (left column) and noise (right column) shocks in the 5-variables VAR. Solid line: point estimate. Grey area: 90% confidence bands.

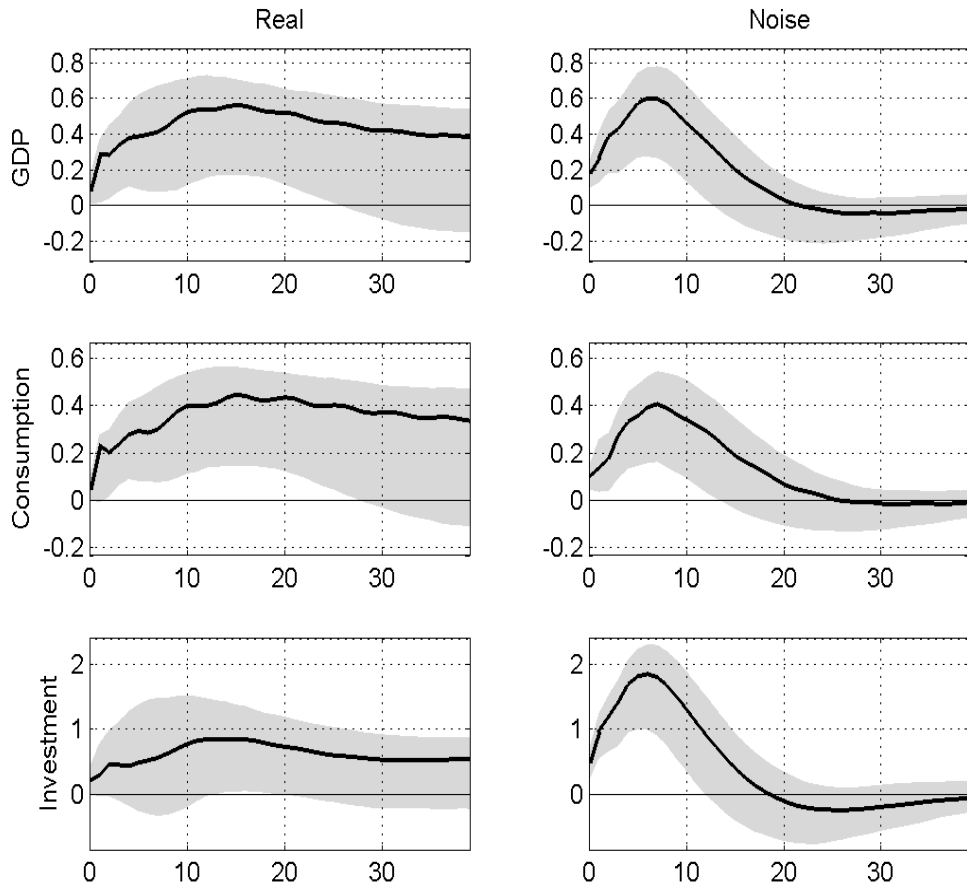


Figure 4: Impulse response functions to real (left column) and noise (right column) shocks in the 5-variables VAR. Solid line: point estimate. Grey area: 90% confidence bands.

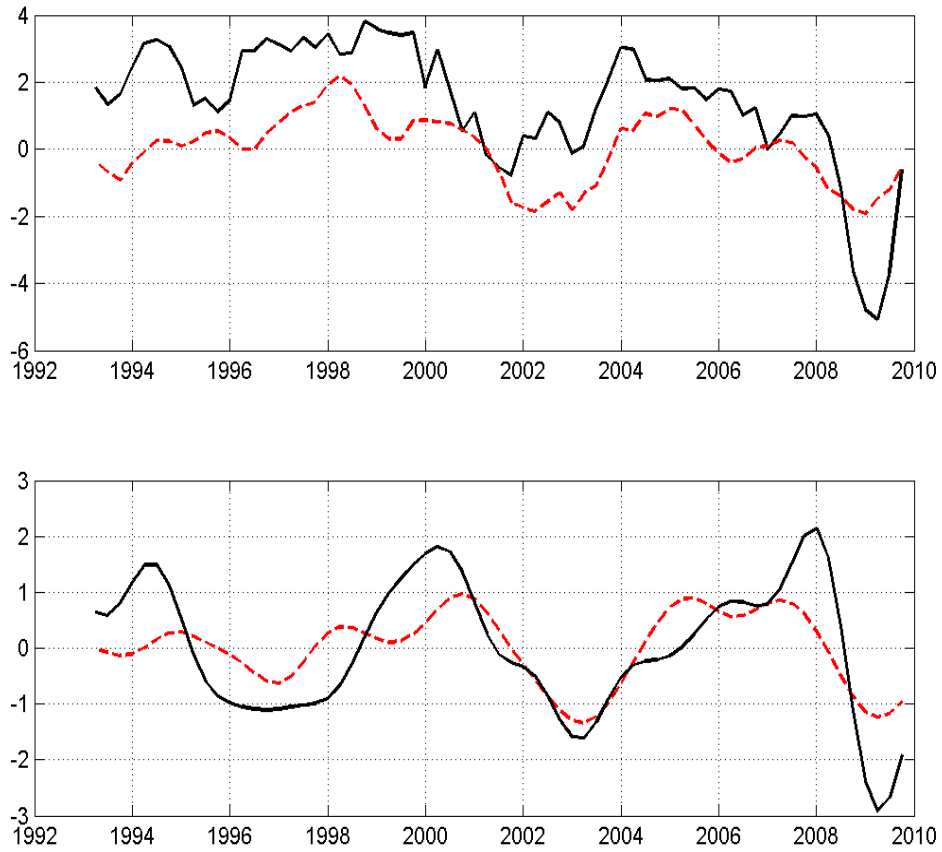


Figure 5: Historical decomposition in the benchmark 5-variables VAR. Top panel. Solid line: yearly growth rates of GDP; dashed line: noise component of the yearly growth rate of GDP. Bottom panel. Solid line: business cycle component of real GDP (frequencies between 6 to 32 quarters); dashed line: noise component of the business cycle component of real GDP.

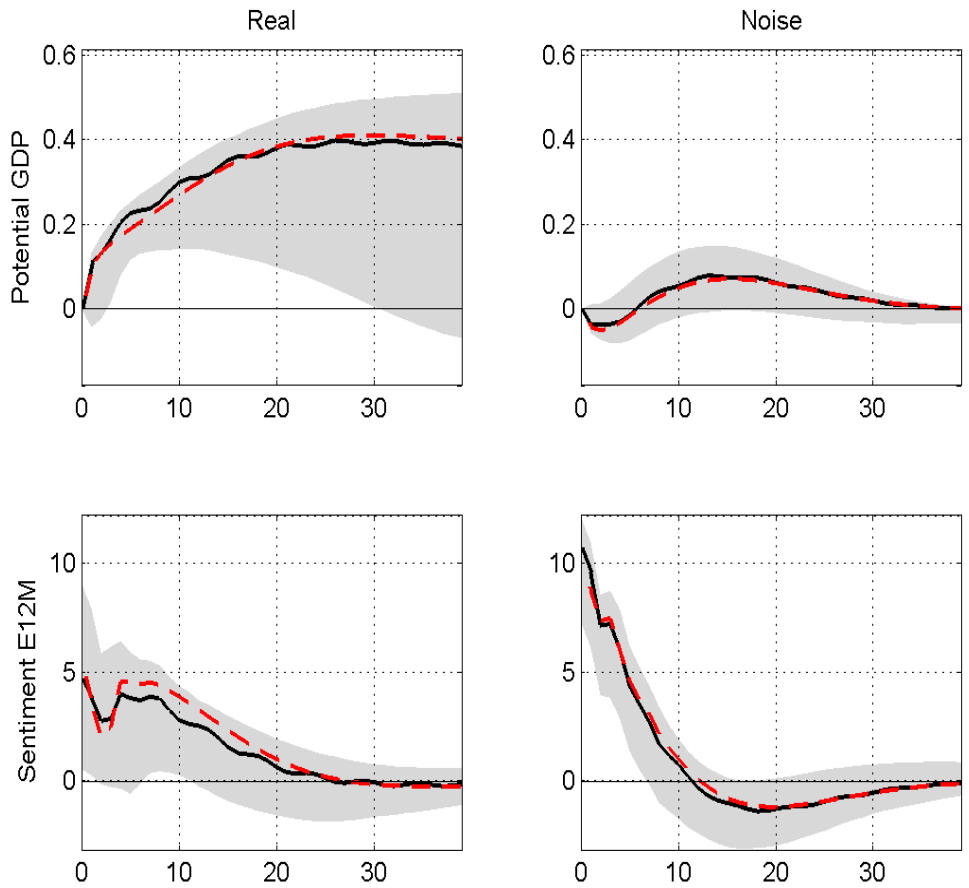


Figure 6: Impulse response functions to real (left column) and noise (right column) shocks in the 5-variables VAR. Solid line: point estimate of the VAR with E12M ordered second. Grey area: 90% confidence bands. Dashed line: point estimate of the VAR with E12M ordered last.



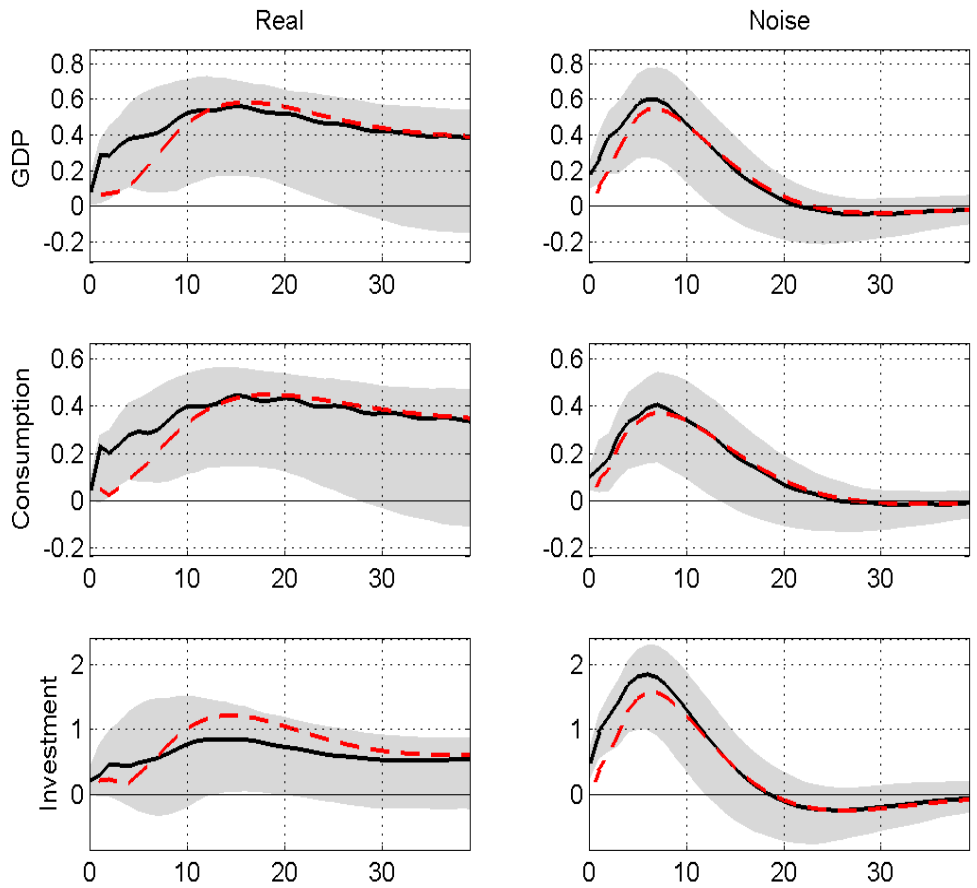


Figure 7: Impulse response functions to real (left column) and noise (right column) shocks in the 5-variables VAR. Solid line: point estimate of the VAR with E12M ordered second. Grey area: 90% confidence bands. Dashed line: point estimate of the VAR with E12M ordered last.

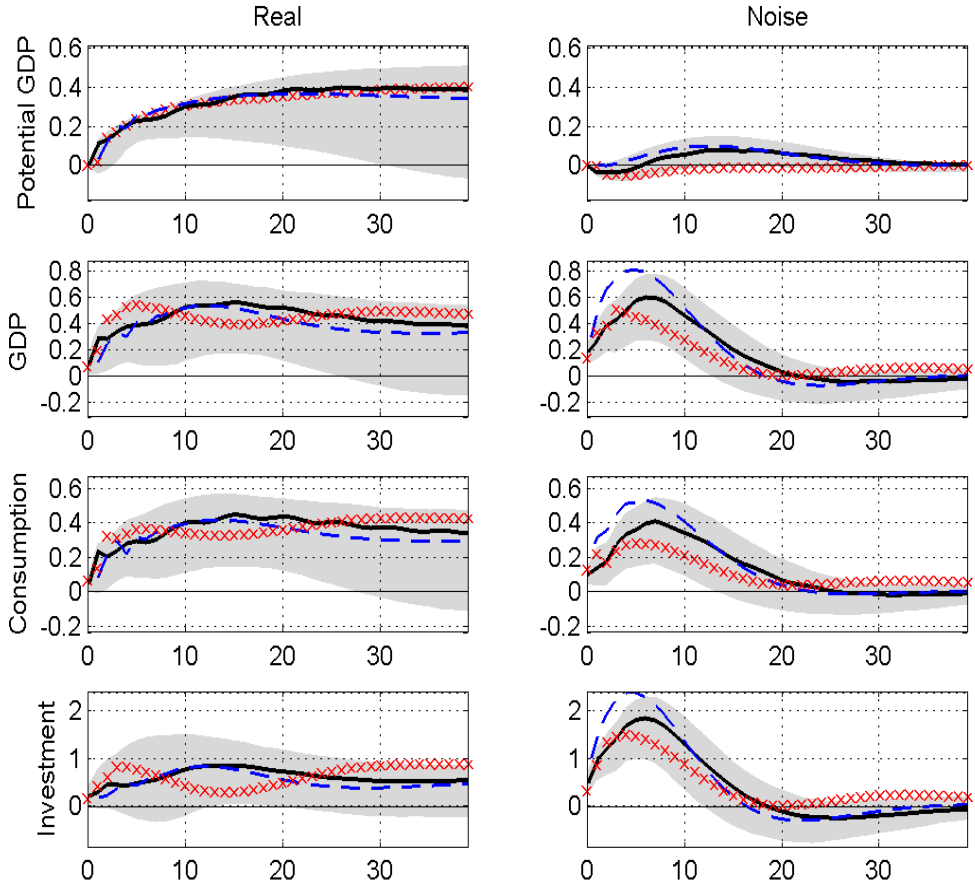


Figure 8: Impulse response functions to real (left column) and noise (right column) shocks in the 5-variables VAR. Solid line: point estimate. Grey area: 90% confidence bands. Dashed line: point estimate of the VAR using the Conference Board Leading Economic Indicators Index as expectation variable. Starred line: point estimate of the VAR using stock prices (S&P500) as expectation variable.

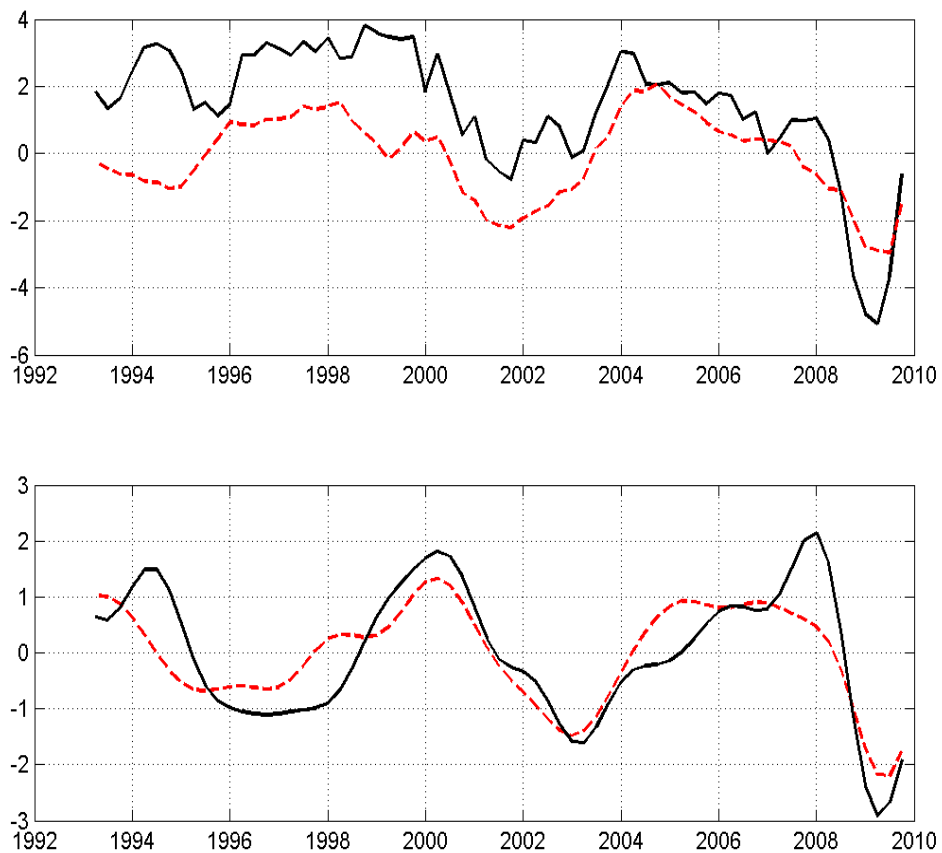


Figure 9: Historical decomposition in the 5-variables VAR using stock prices (S&P500) as expectation variable. Top panel. Solid line: yearly growth rates of GDP; dashed line: noise component of the yearly growth rate of GDP. Bottom panel. Solid line: business cycle component of real GDP (frequencies between 6 to 32 quarters); dashed line: noise component of the business cycle component of real GDP.

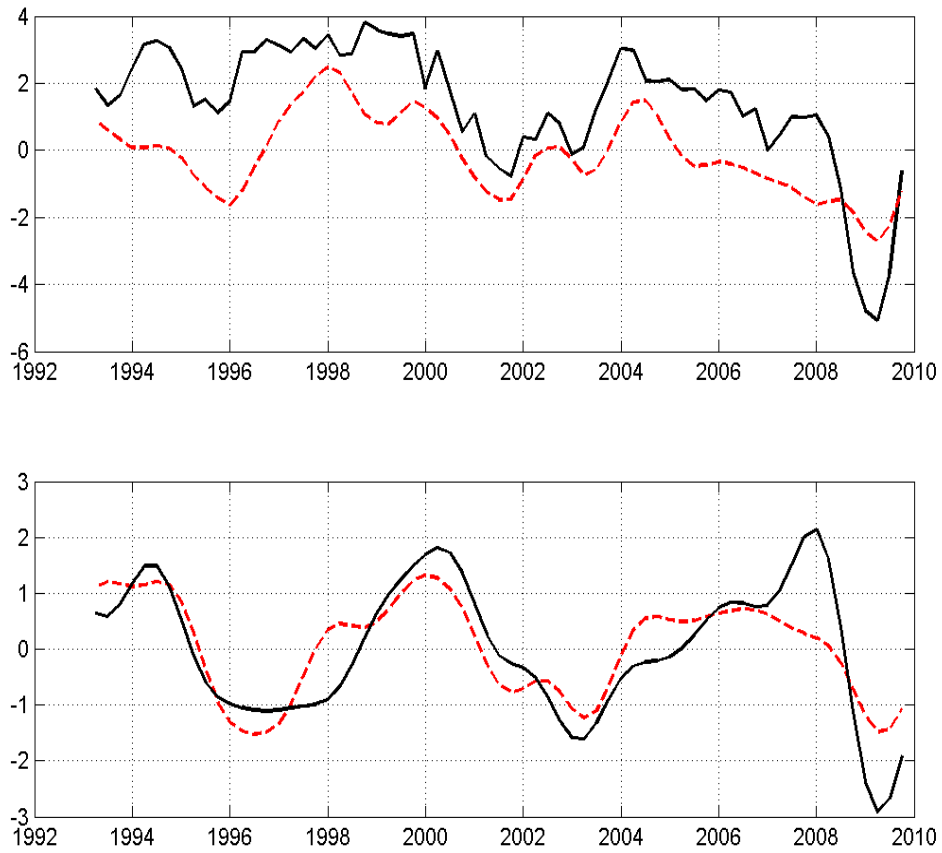


Figure 10: Historical decomposition in the 5-variables VAR using the Conference Board Leading Economic Indicators Index as expectation variable. Top panel. Solid line: yearly growth rates of GDP; dotted line: noise component of the yearly growth rate of GDP. Bottom panel. Solid line: business cycle component of real GDP (frequencies between 6 to 32 quarters); dotted line: noise component of the business cycle component of real GDP.